Considering Transient Effect in Spectrum Analysis

LAC 2009

April 18th, 2009

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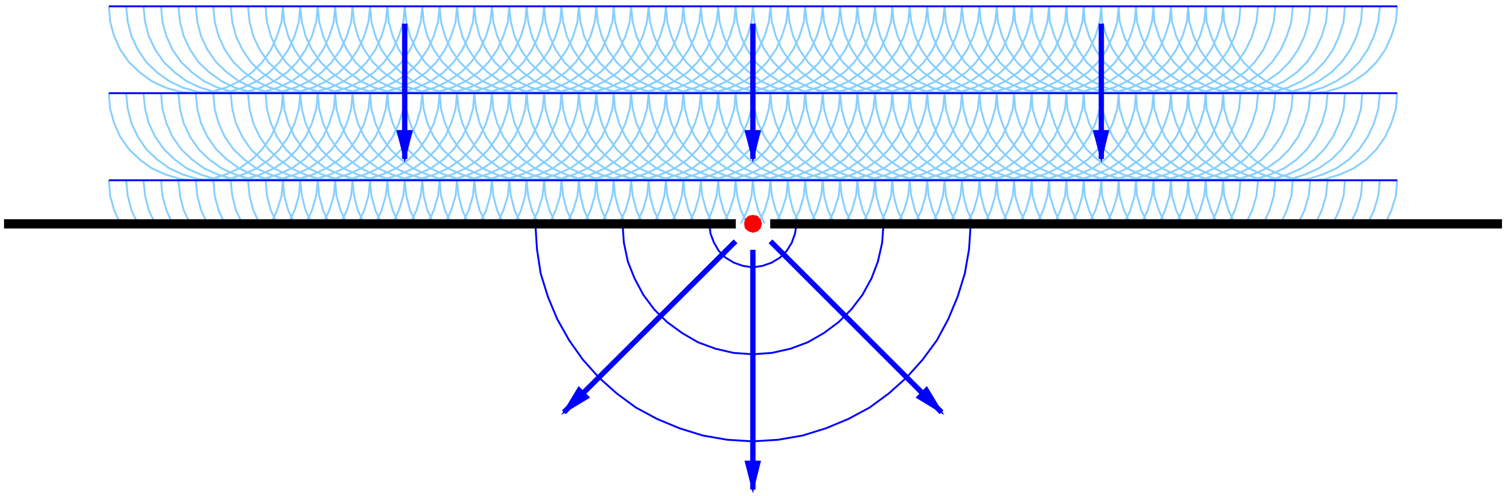
The Idea

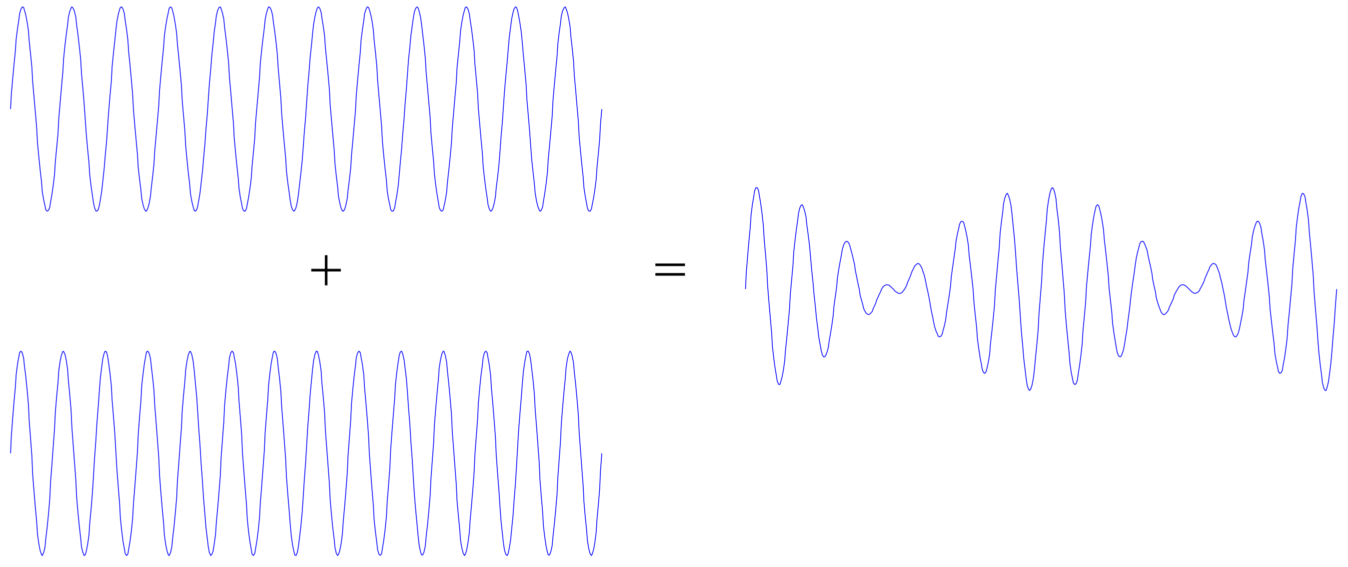
- van Huygens principle
- superposition principle

→ decomposition into elementary waves



The Problem

- Ambiguity: $\cos(x) + \cos(y) = 2\cos(\frac{x-y}{2})\cos(\frac{x+y}{2})$



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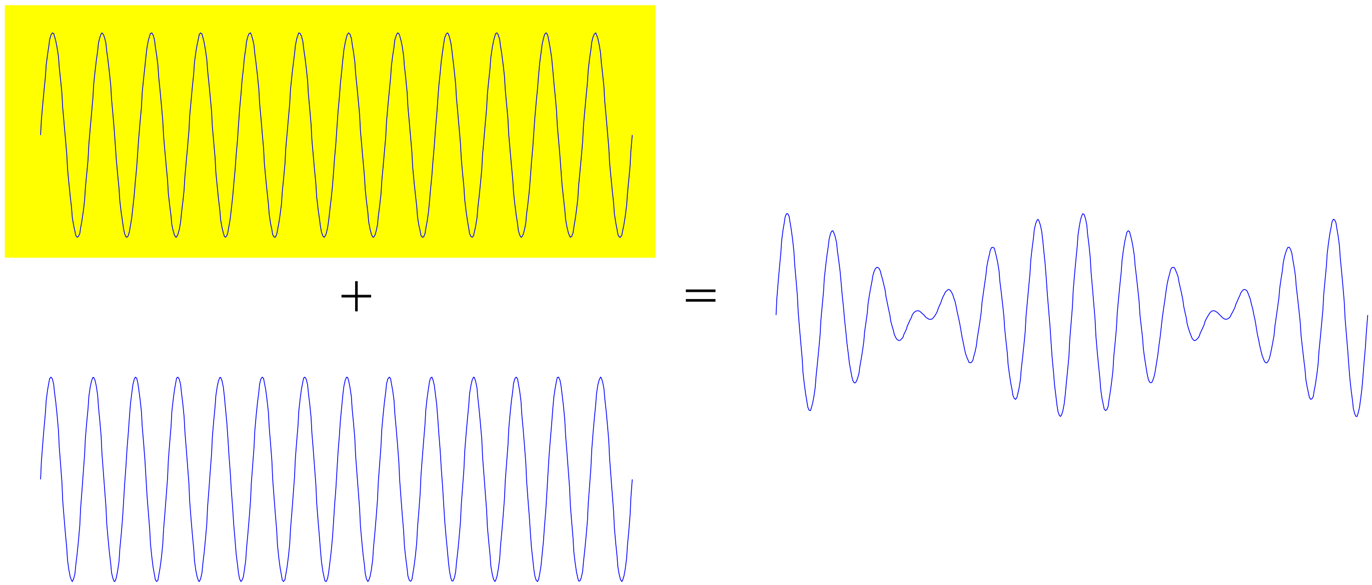

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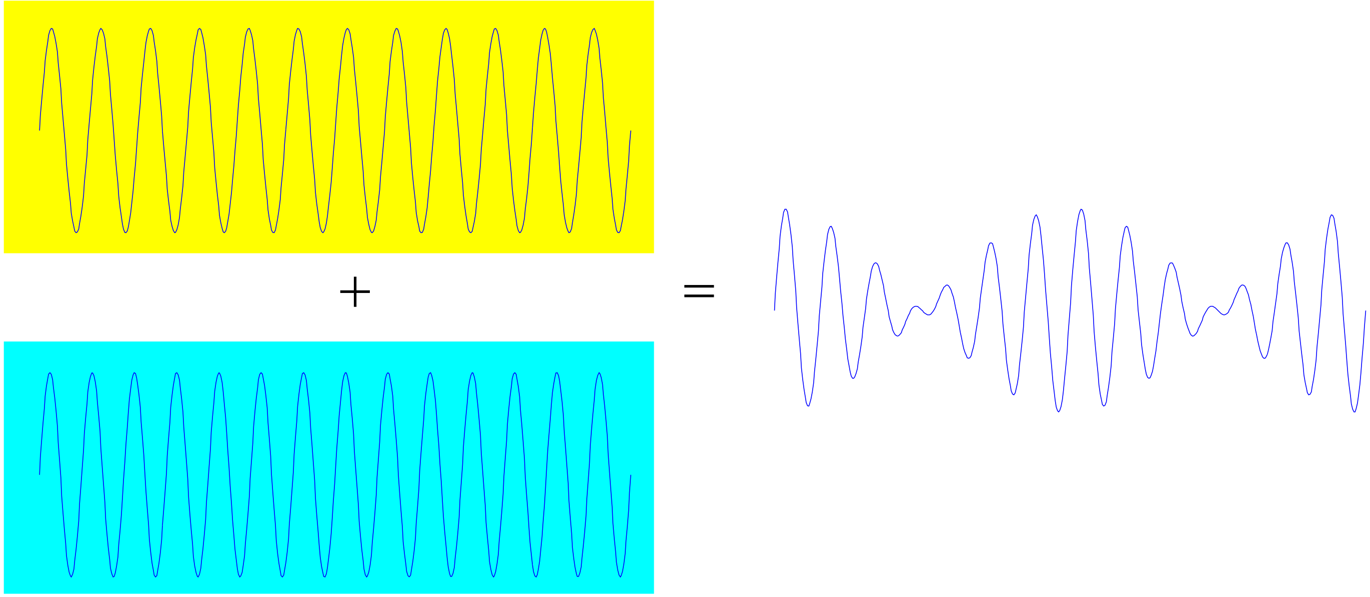

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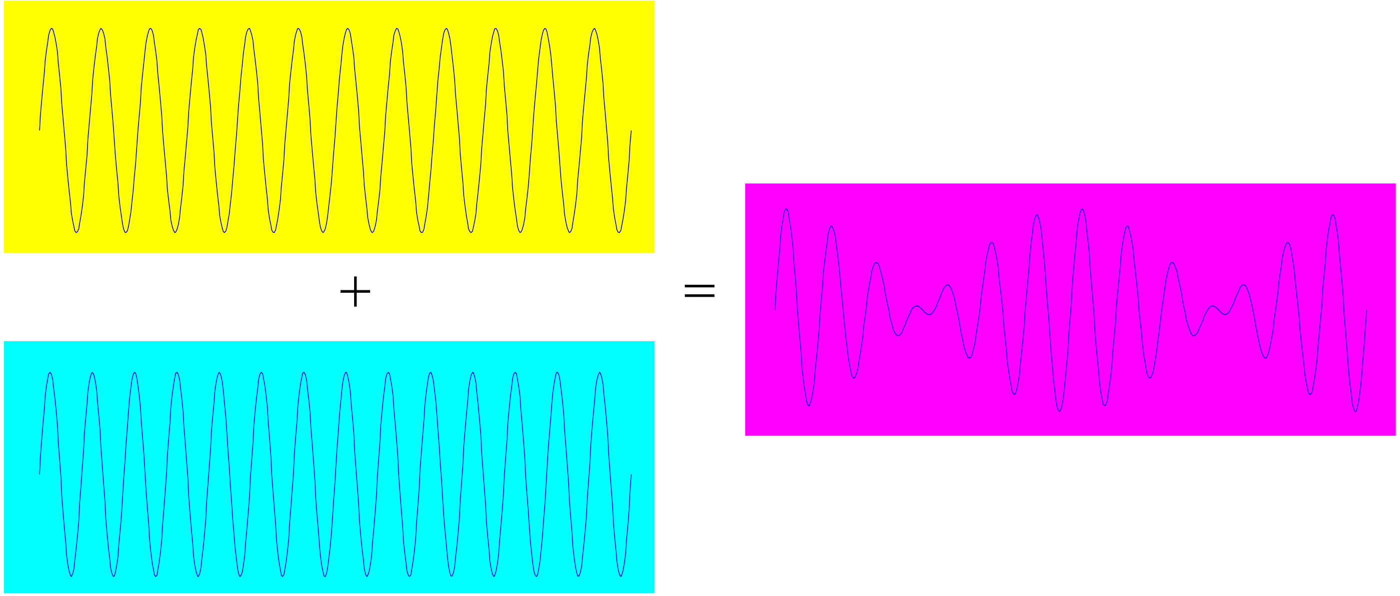

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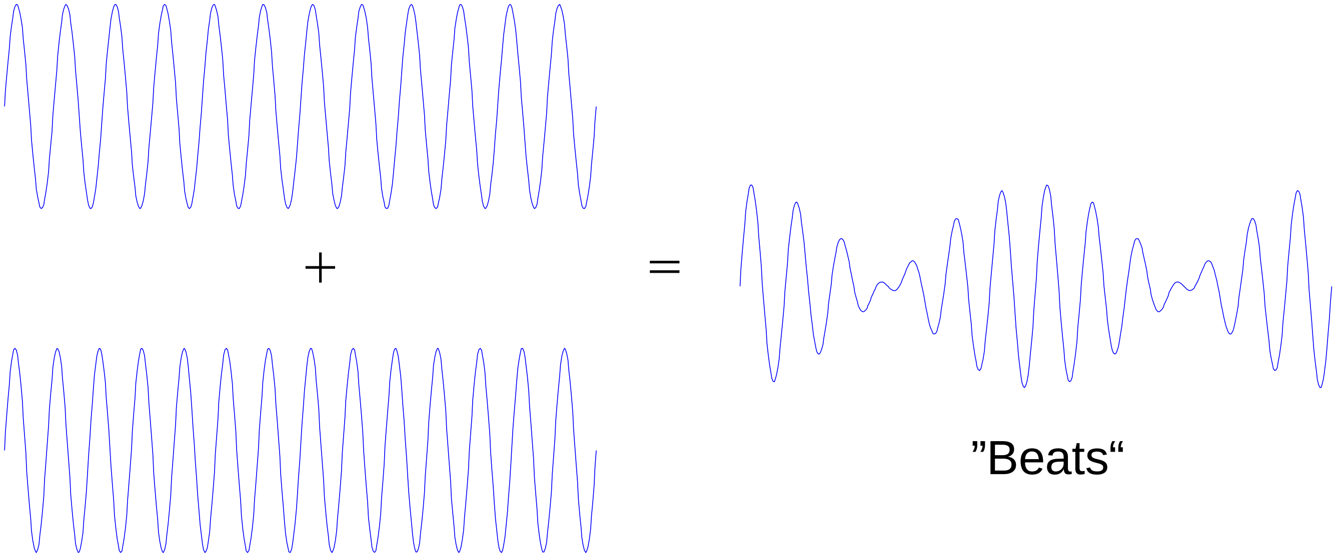
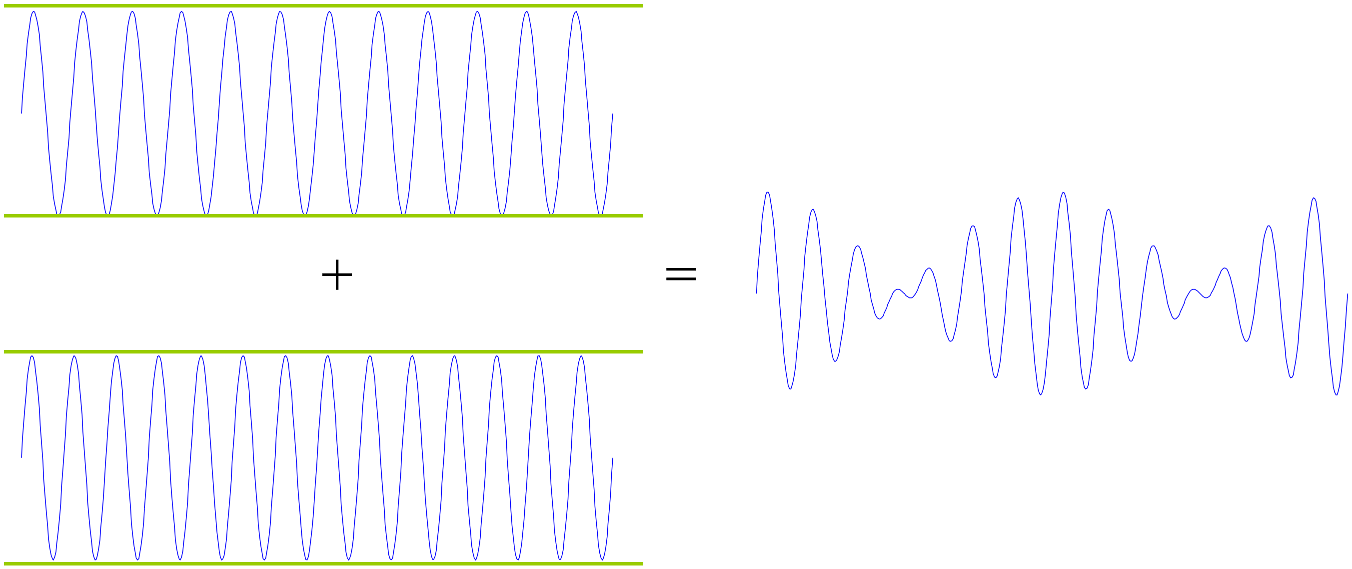


”Beats“

The Problem

- Ambiguity: $\cos(x) + \cos(y) = 2\cos(\frac{x-y}{2})\cos(\frac{x+y}{2})$

time-invariant amplitude



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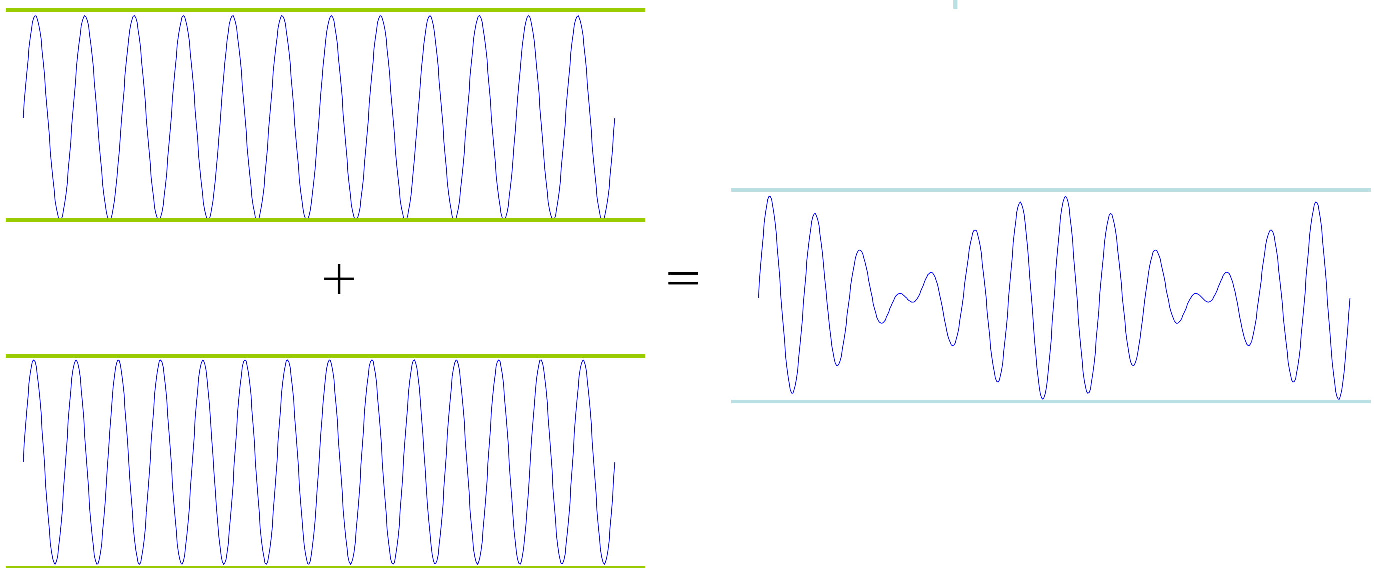
time-invariant amplitude

time-varying amplitude



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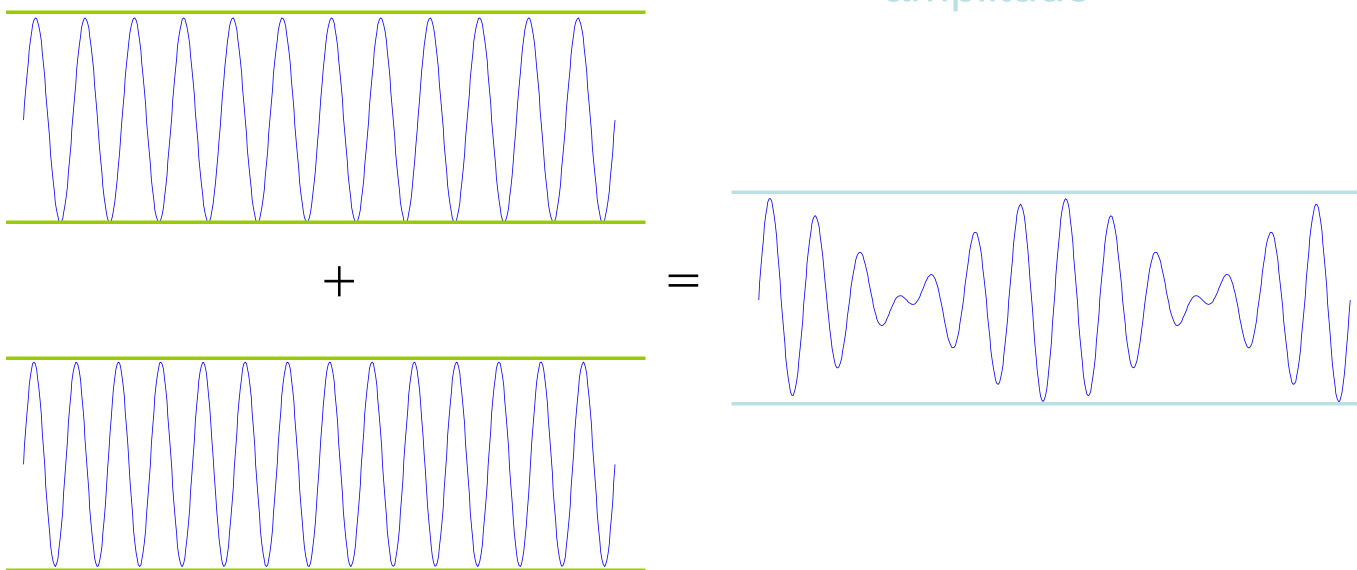
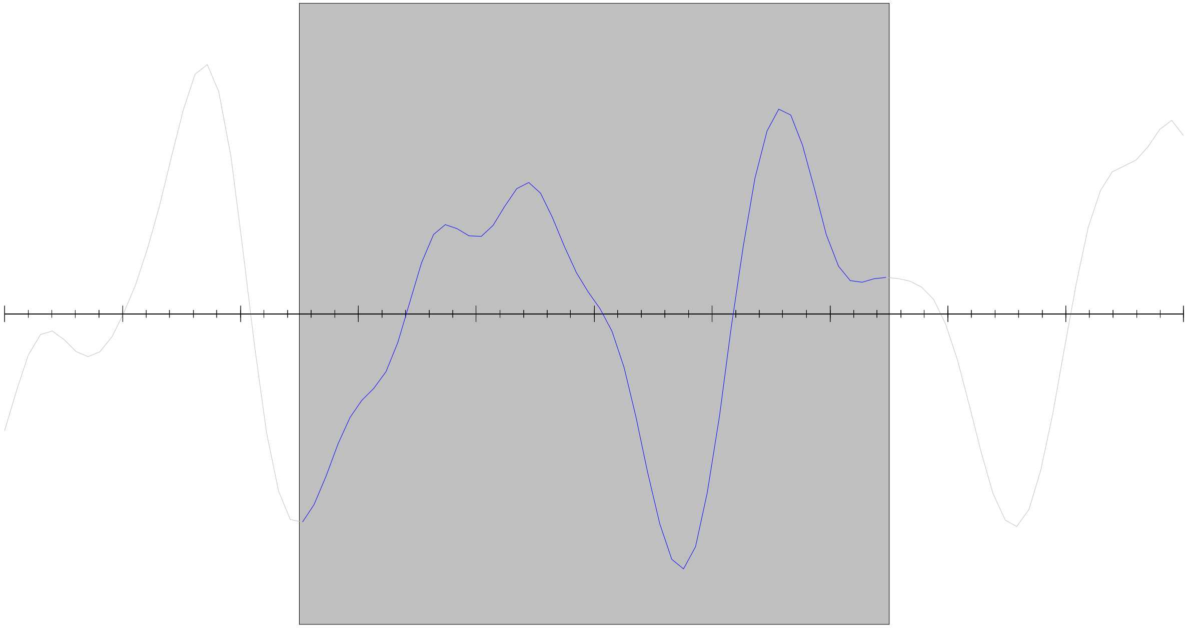


Fourier ???

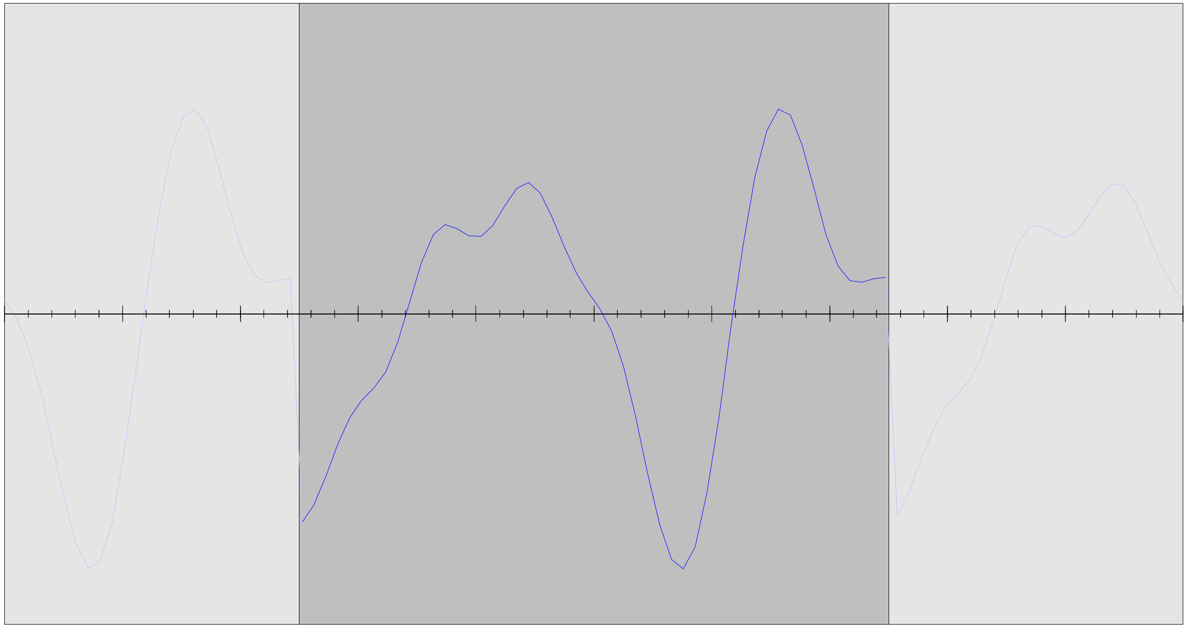
State of the Art

original signal – selected window



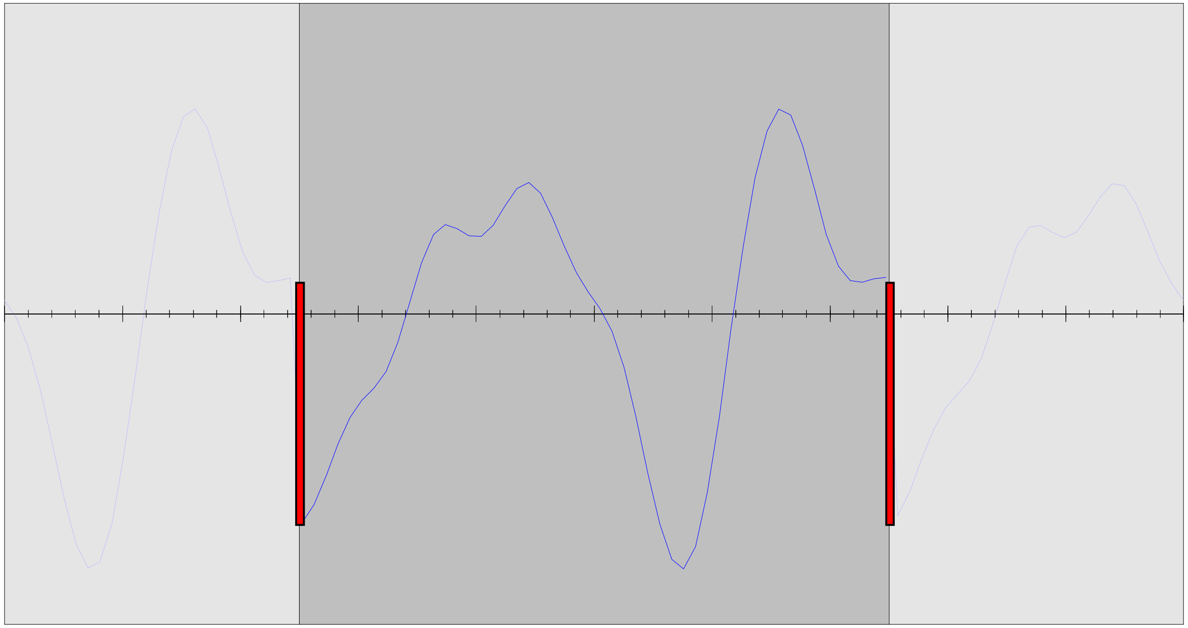
State of the Art

periodic continuation



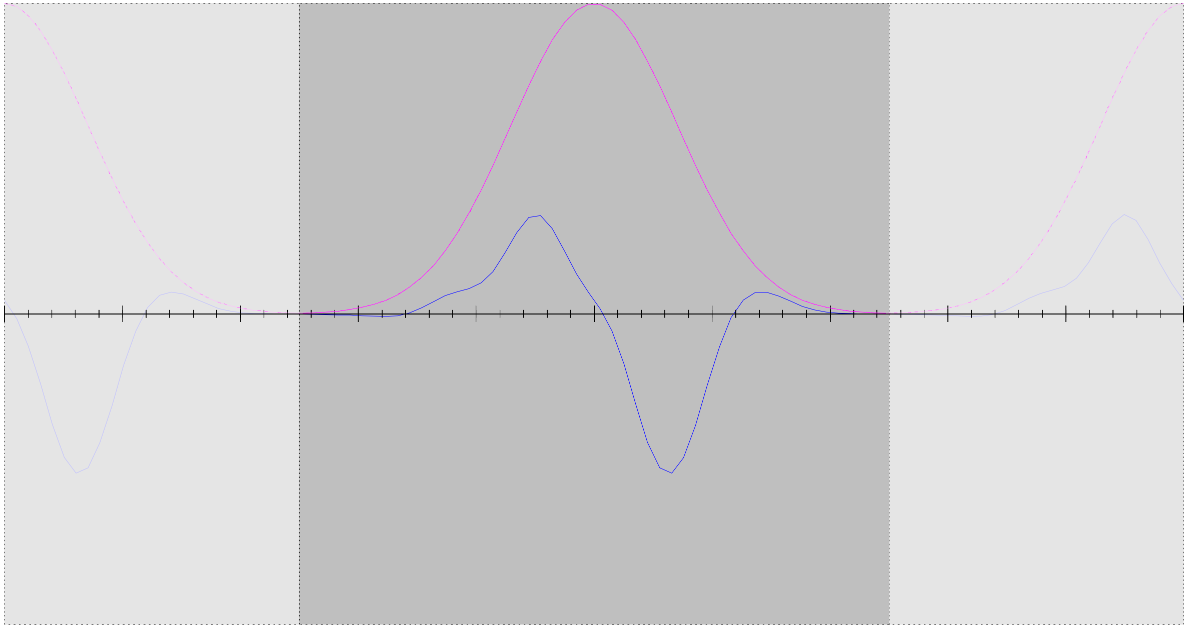
State of the Art

discontinuity in
periodic continuation
→ faulty high frequency peak



State of the Art

avoid discontinuity
by Gaussian filter



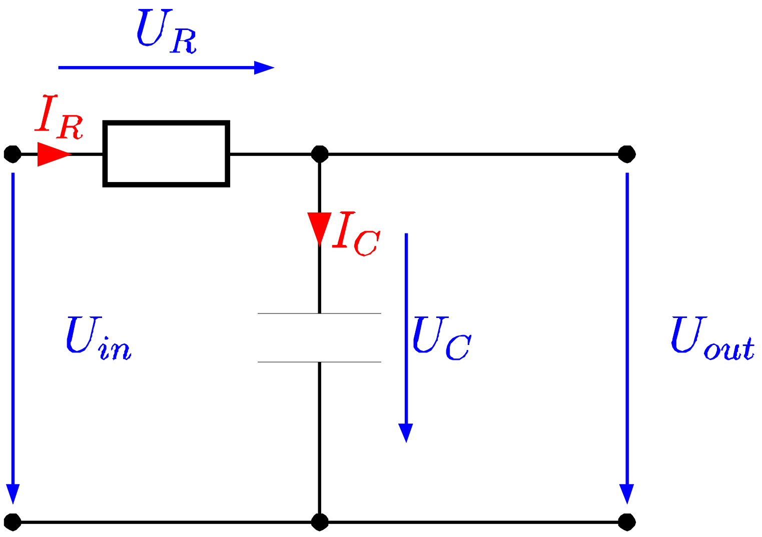
State of the Art

- Gaussian filter changes signal
- signal response delayed by Gaussian „fade in“

→ desire for alternate with immediate response

Case Study

- RC Low Pass filter
- probably simpliest linear filter



Case Study

- RC Low Pass Response function:

$$\mathrm{U_{out}}(t_1) = e^{\frac{t_0 - t_1}{\tau}} \mathrm{U_{out}}(t_0) + \frac{1}{\tau} \int_{t_0}^{t_1} \mathrm{U_{in}}(t) e^{\frac{t - t_1}{\tau}} \mathrm{d}t$$

or

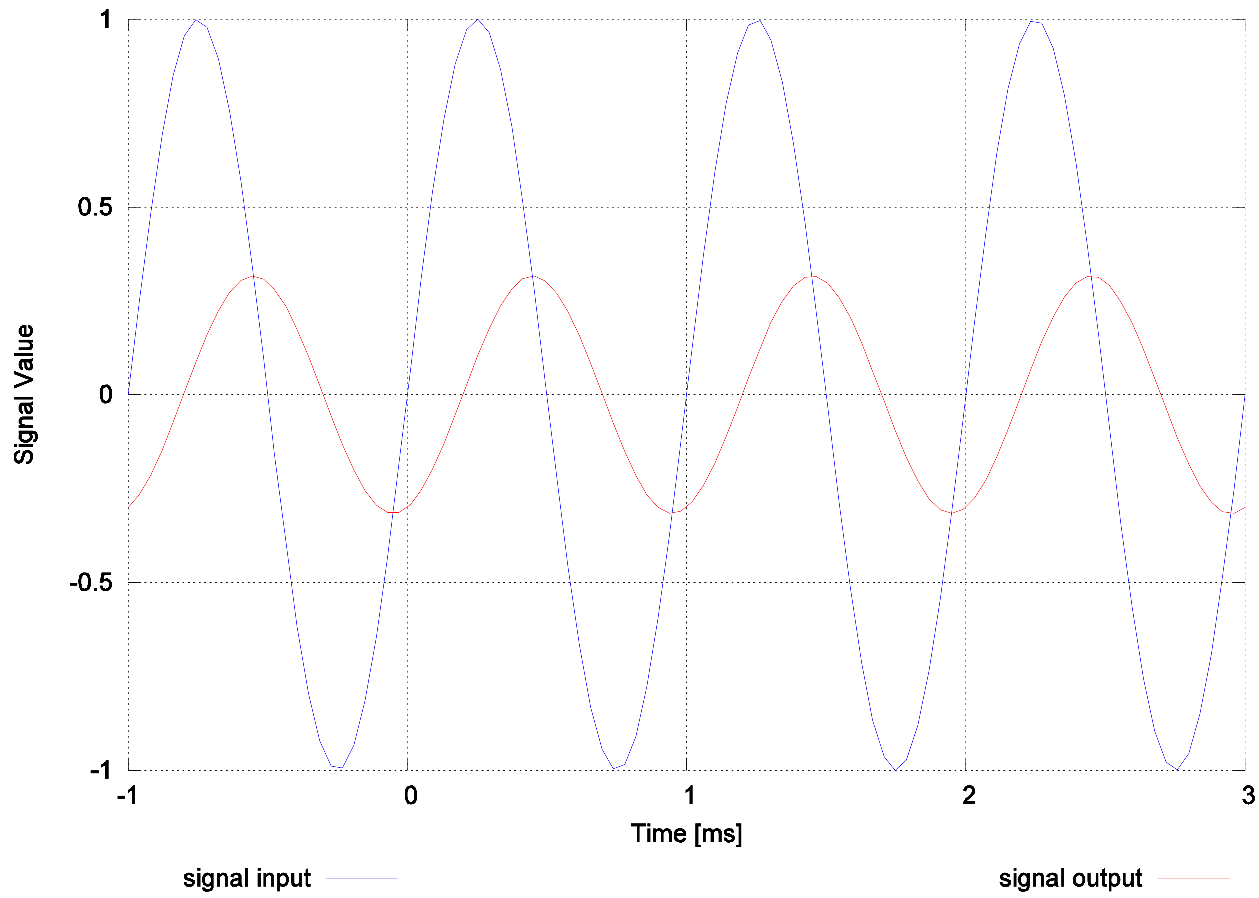
$$\mathrm{U_{out}}(t_1) = \frac{1}{\tau} \int_{-\infty}^{t_1} U_{\mathrm{in}}(t) e^{\frac{t - t_1}{\tau}} \mathrm{d}t$$

- $U_{Out}(t_0)$ and signal history between (t_0, t_1) is sufficient for computing $U_{Out}(t_1)$

Case Study

- Steady State Transfer

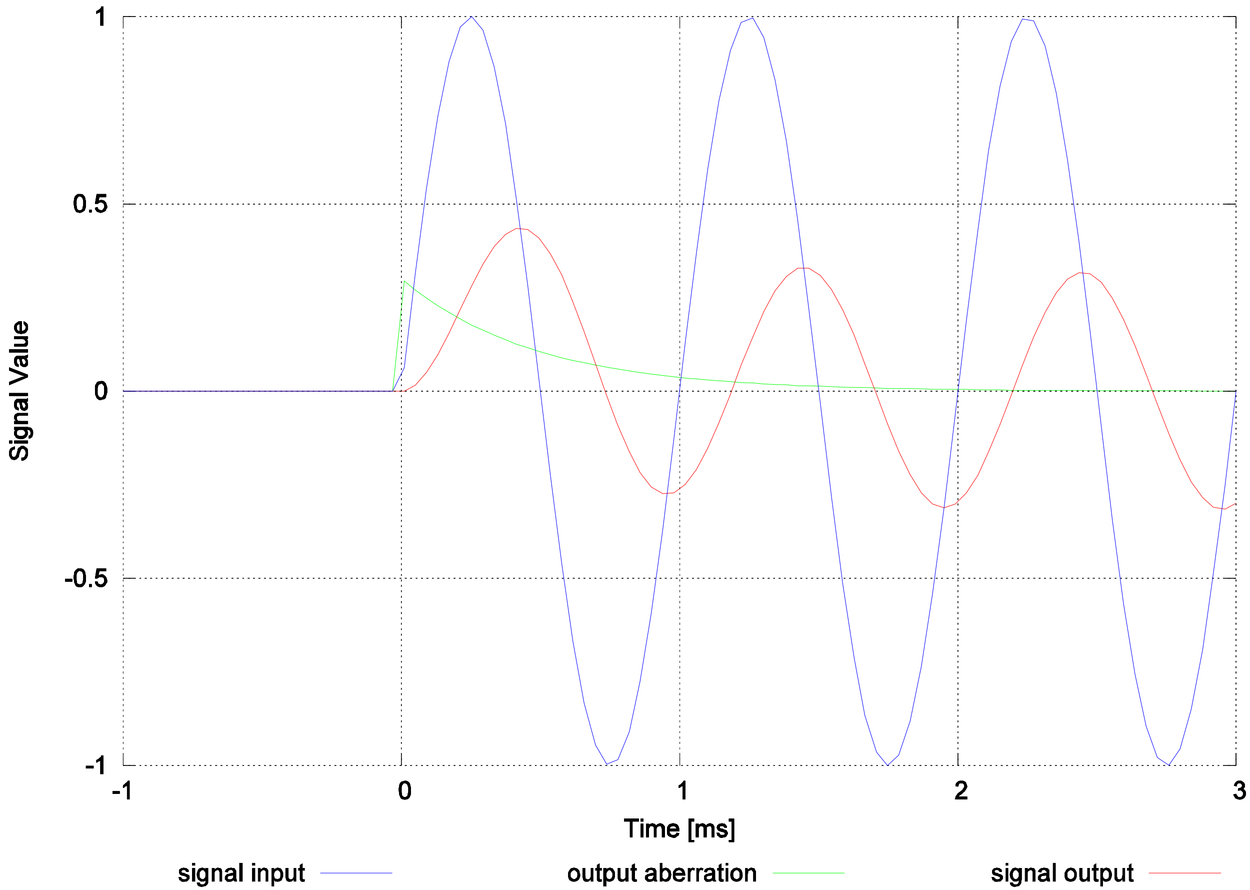
$$\mathrm{U_{out}}(t_1) = \frac{e^{-i\arctan(\phi\tau)}}{\sqrt{1+(\phi\tau)^2}}\mathrm{U_{in}}(t_1)$$



Case Study

- Transient Effect

$$U_{\text{out}}(t) = \begin{cases} \frac{\sin(t) - \tau\cos(t) + \tau e^{\frac{-t}{\tau}}}{\tau^2+1} & \forall t \geq 0, \\ 0 \text{ otherwise.} \end{cases}$$



Observations

RC Low Pass Response

$$\mathrm{U}_{\mathrm{out}}(t_1) = \frac{1}{\tau} \int_{-\infty}^{t_1} U_{\mathrm{in}}(t) e^{\frac{t-t_1}{\tau}} \mathrm{d}t$$

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$$\mathrm{U}_{\mathrm{out}}(t_1) = \frac{1}{\tau}\int_{-\infty}^{t_1} U_{\mathrm{in}}(t)\, e^{\frac{t-t_1}{\tau}}\,\mathrm{d}t$$

time as parameter

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time as parameter

past time only contribution

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time as parameter

past time only contribution

exponential signal decay

Spectral Transformation (”ST“)

$$\mathcal{S}(f(t), \phi, t_0) := (\mu - 2\pi i)\phi \int_{-\infty}^{t_0} f(t) e^{(\mu - 2\pi i)\phi(t - t_0)} \mathrm{d}t$$

FT versus ST

$$\mathcal{F}(f(t),\phi) := \int_{-\infty}^{+\infty} f(t) e^{-2\pi i \phi t} \mathrm{d}t$$

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ST versus LT

$$\mathcal{L}(g(r), s) = \int_0^\infty e^{-sr} g(r) \mathrm{d}r$$

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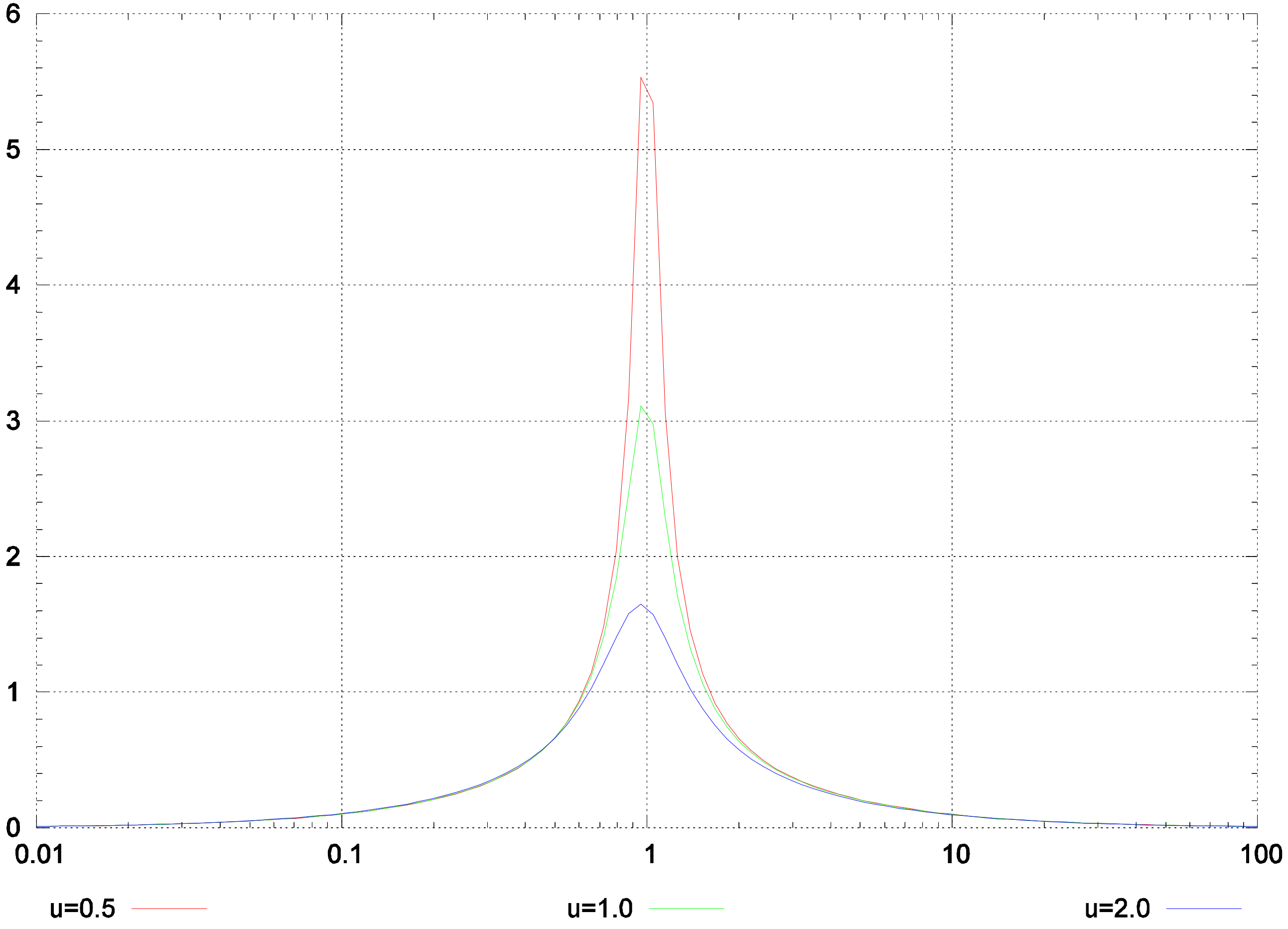
$$s\mathcal{L}(g(r), s) = \mathcal{S}(f(t), \phi)$$

for $s = (\mu - 2\pi i)\phi$, $r = -t$, and $g(r) \equiv f(t)$

ST Sine Transfer

$$\mathcal{S}(\sin(\alpha t), \phi) = \frac{-(\mu - 2\pi i)\phi\alpha}{(\mu - 2\pi i)^2\phi^2 + \alpha^2}$$



ST Recursive Form

$$\begin{aligned}\mathcal{S}(f(t),\phi,t_1) =& e^{-(\mu-2\pi i)\phi\Delta t}\mathcal{S}(f(t),\phi,t_0)+\\ &(1-e^{-(\mu-2\pi i)\phi\Delta t})f(t_0)\end{aligned}$$

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For comparison – Sliding Window DFT:

$$F_{t+1}(n) = (F_t(n) - f_t + f_{t+N})e^{2\pi i\frac{n}{N}}$$

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infinite fade out vs. fall-off from window

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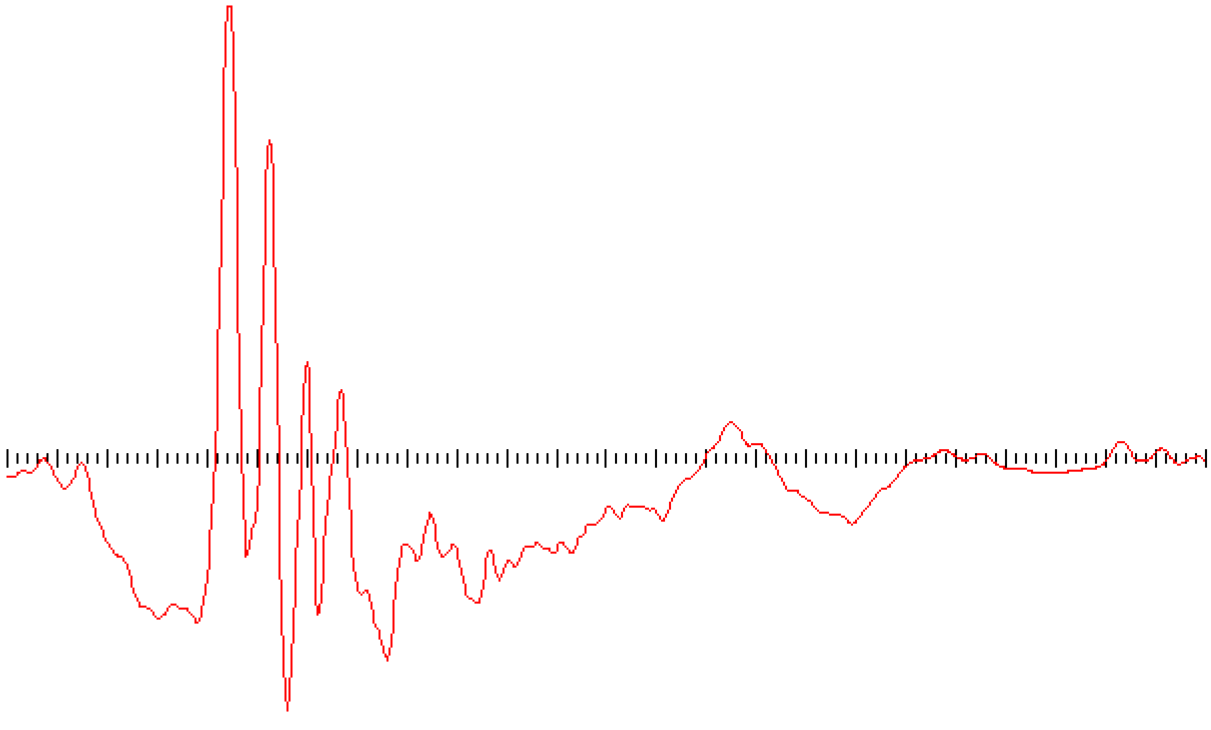
infinite fade out vs. fall-off from window

open left-hand window vs. fixed-size window

Evaluation

Test signal: a slap noise



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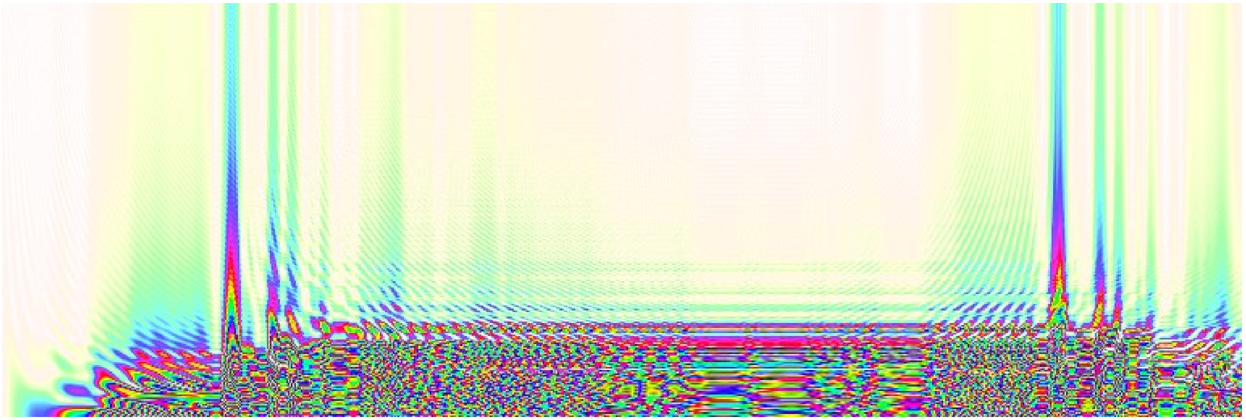
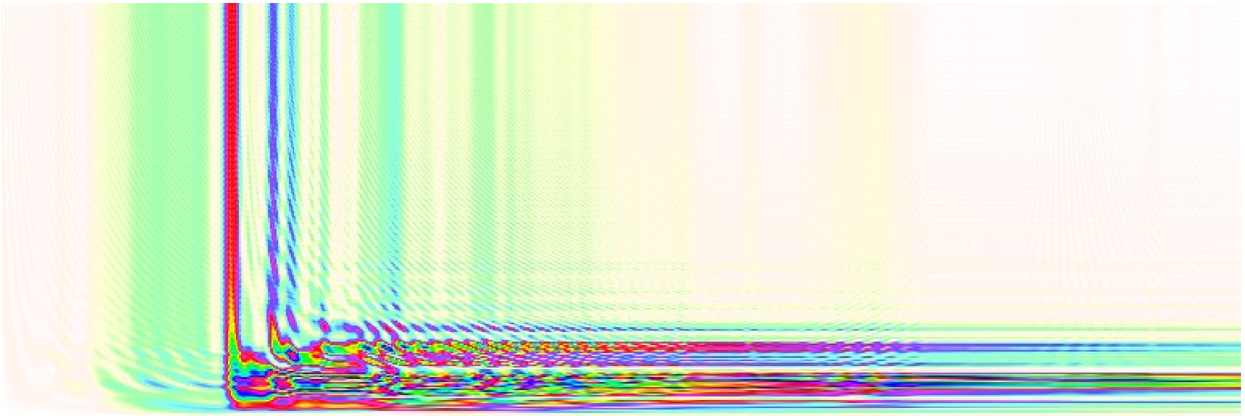
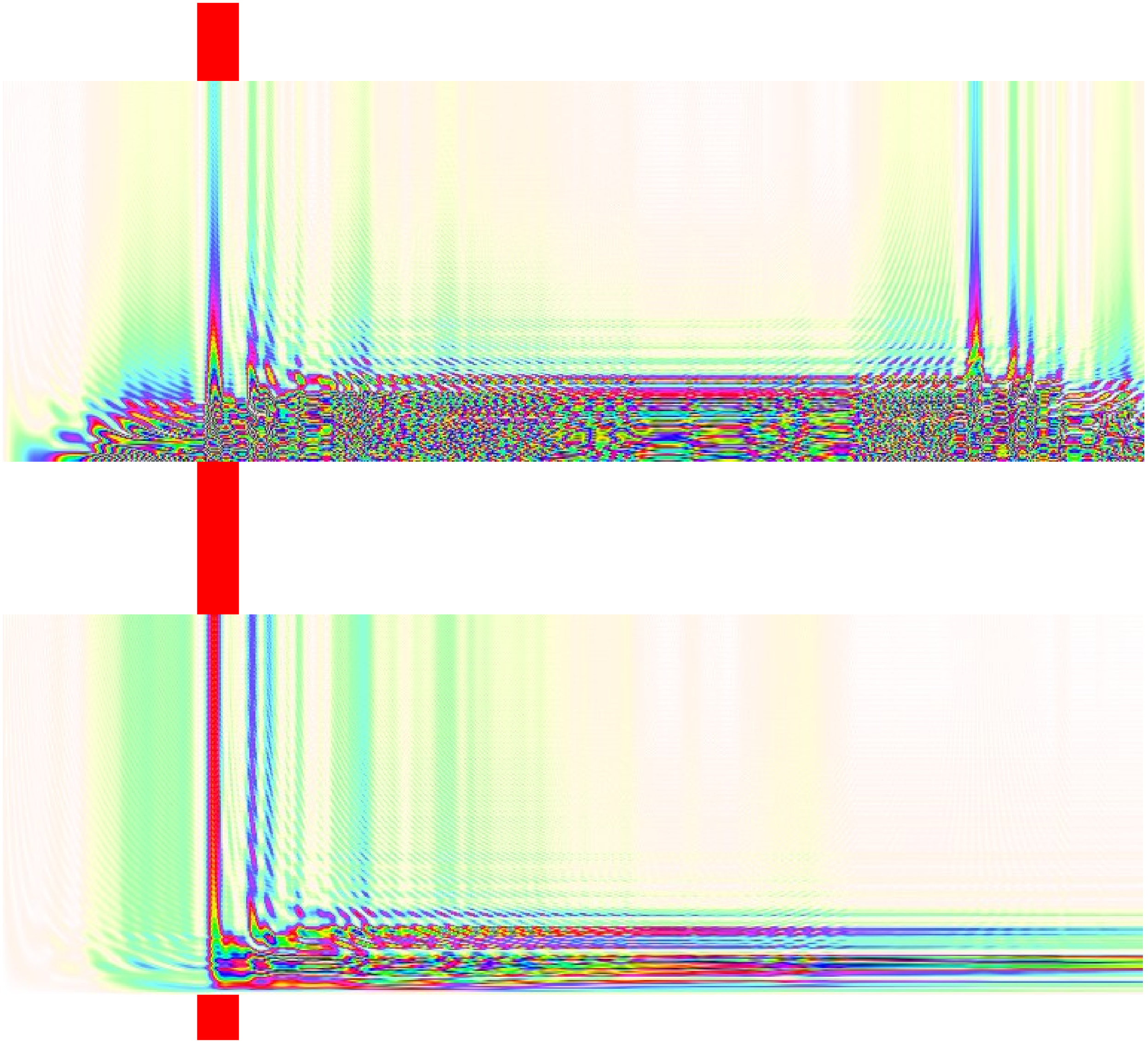


slap noise DFT



slap noise DST

Evaluation

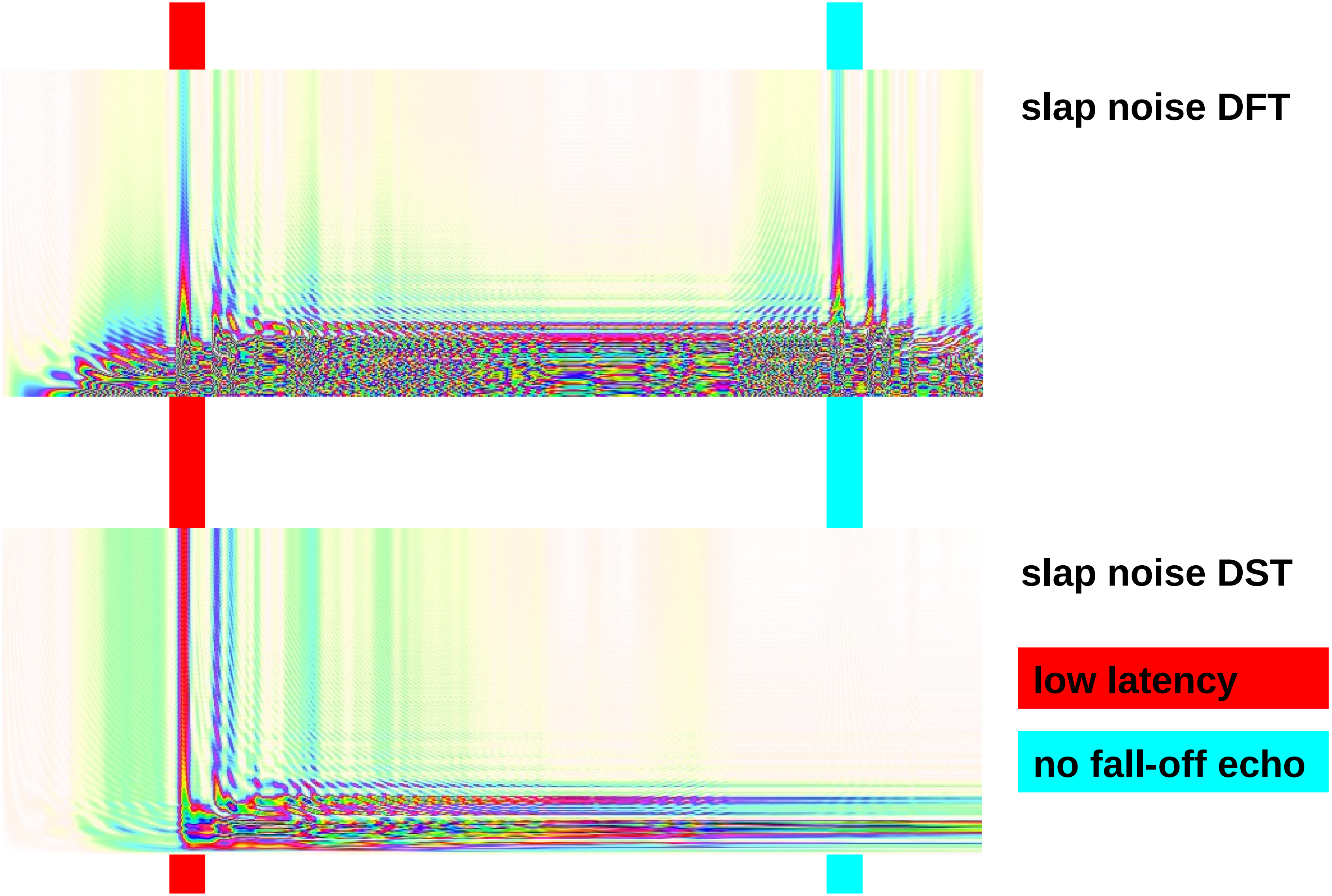


slap noise DFT

slap noise DST

low latency

Evaluation



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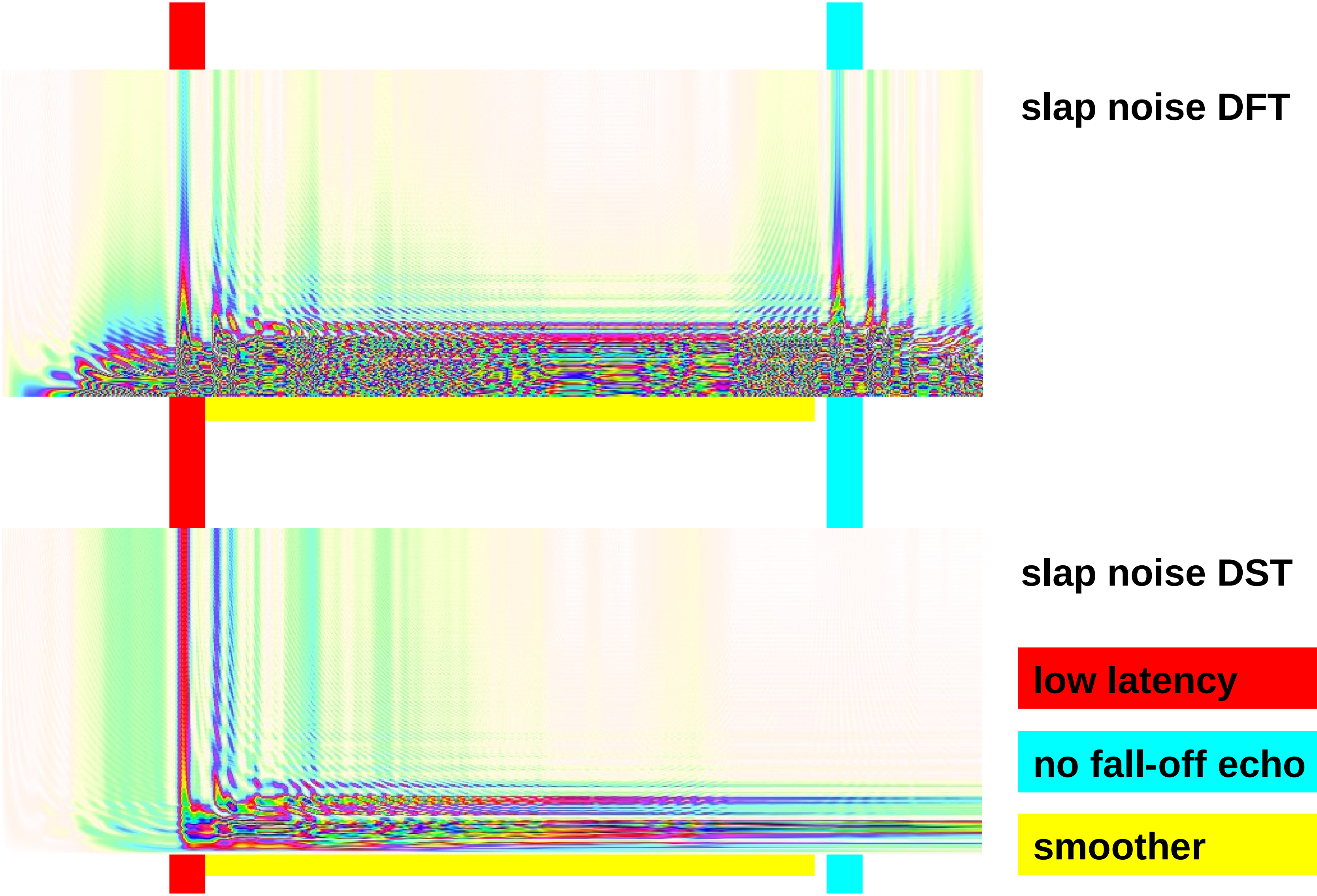
slap noise DFT
slap noise DST
low latency
no fall-off echo
smoother

Future Work

- discrete back transformation
- practical applications
- logarithmically sized frequency lines

Conclusion

- developed ST as alternative to FT
- strongly related to LT
- features transient effect
- low latency
- smoother / better stability
- practical back transformation still missing

Questions?