Muditulib
a multi-dimensional tuning library

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http://muditulib.eu

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Tuning of pitch within the digital domain
1. Tuning theory
2. Music theory: the diatonic scale
3. Multi-dimensional pitch representation systems
4. $Ts()$: a 2-dimensional tuning system
5. $Tts()$: a 3-dimensional tuning system
6. Implementation of the software
Tuning theory

2 Music theory: the diatonic scale
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Equal temperament (12-TET)

\[ f = 2^{(m-69)/12} \cdot 440 \]

Harmonic spectrum:

- **partials** 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : etc..

Intervals:

- **octave**: \( 2^{(12/12)} = 2 \)
- **perfect fifth**: \( 2^{(7/12)} \neq \frac{3}{2} \)
- **major third**: \( 2^{(4/12)} \neq \frac{5}{4} \)
Tuning theory

Music theory: the diatonic scale

Multi-dimensional pitch representation systems

$Ts()$: a 2-dimensional tuning system

$Tts()$: a 3-dimensional tuning system

Implementation of the software
The diatonic scale

Consists of two variable steps:

- The whole tone ($T$)
- The semitone ($s$)

The frequency ratio between the eighth and the first note - generally 2 : 1 - is called the octave ($x$).

$$x = T^5 \cdot s^2$$
The diatonic scale

The tonal system used as a starting point is a 7-tone and NOT a 12-tone system.
1 Tuning theory
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3 **Multi-dimensional pitch representation systems**
4 $Ts()$: a 2-dimensional tuning system
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Traditional music notation

- Traditional music notation is based on seven notes and the two kinds of steps, and is therefore two-dimensional.
- All intervals in diatonic music are built up from a number of whole tones and a number of semitones.
Two-dimensional representation of western note naming system.

One horizontal step is one whole tone, one vertical step a semitone.
Example intervals

Intervals can be expressed as a list of two values

octave: \( T^5 \cdot s^2 \rightarrow (5, 2) \)

perfect fifth: \( T^3 \cdot s \rightarrow (3, 1) \)

major third: \( T^2 \rightarrow (2, 0) \)

minor third: \( T \cdot s \rightarrow (1, 1) \)
Example pitches

Pitches can be expressed as a list of two values (as a distance to a reference pitch) as well.

The reference pitch is set equal to the $C$ corresponding to MIDI note 0. Such a pitch will be called a $Ts()$ value, containing $T_n$ and $s_n$, the number of whole tones and semitones, respectively.

- $C, C(0,0)$
- $C', Ts(25, 10)$
- $B\#$ (german: $H\#$) $Ts(26, 8)$

$Ts()$ values can be easily translated to MIDI notes.

$$m = 2 \cdot T_n + s_n$$
The total amount of steps \((T_n + s_n)\) defines the character (one of seven \(((T_n + s_n) \% 7))\).

Flats or sharps can be extracted from the difference in number between whole tones and semitones.
Obtaining $T_s()$ values

- Directly from the composition (e.g. Lilypond to $T_s()$)
- User-defined: ‘mod’ parameter
- Algorithmic: real-time pitch spelling
The modulation parameter

The reference is the standard baroque keyboard setting:

Eb, Bb, F#, C#, G#

Every modulation up replaces one note in the circle of fifths by adding \((1, -2)\) to its assigned \(T_S\) value, starting at MIDI note 3 (E-flat to D-sharp). In the opposite direction it starts at MIDI note 8 (G-sharp to A-flat). In the current implementation each modulation change is calculated from a default array at ‘mod 0’.
Figure: Examples of [midi2ts] and [ts2symbol] in Pure Data.
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Two-dimensional pitches enable a two-dimensional tuning

Examples of two-dimensional tunings/temperaments are

- Pythagorean
- Mean tone

Temperaments that can be one- AND two-dimensional are the \( n \)-TET temperaments.

- 12-TET (2-1)
- 19-TET (3-2)
- 31-TET (5-3)
- 53-TET (9-4; can even be three-dimensional (\( Tts \), 9-8-5))
The type of a two-dimensional tunings can be defined by ratio $r$.

$r$ is the semitone to whole tone ratio.

$$r = \frac{\log s}{\log T}$$

Pythagorean (perfect fifth $(3, 1)$ and octave $(5, 2)$):

- $T = \frac{9}{8}$, $s = \frac{256}{243}$
- $r \approx 0.44247$

$$\frac{3}{2} = 2^{(3+r)/(5+2r)}$$
Other two-dimensional examples

Mean tone (major third $(2, 0)$ and octave $(5, 2)$): $r \approx 0.60628$

\[ \frac{5}{4} = 2^{2/(5+2r)} \]

$n$-TET:

- 12-TET $r = \frac{1}{2}$
- 19-TET $r = \frac{2}{3}$
- 31-TET $r = \frac{3}{5}$
- 53-TET $r = \frac{4}{9}$
Calculate $Ts()$ to frequency $f$

$A$ is the reference frequency at $Ts(29, 11)$ in cycles per second.

$$f = A \left( (T_n - 29) + (s_n - 11) \cdot r \right) / (5 + 2 \cdot r) \cdot A$$
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Adding a third dimension enables five-limit just intonation and Turkish modes.
The whole tones are then divided into a number of whole tones ($T$) and some slightly smaller whole tones ($t$).

Some examples:

- octave: $(3, 2, 2)$
- perfect fifth: $(2, 1, 1)$
- major third: $(1, 1, 0)$
- minor third: $(1, 0, 1)$
The difference between $T$ and $t$ in five-limit just intonation ($\frac{81}{80}$) is called the *syntonic comma*. $A$ is the reference at $Tts(17, 12, 11)$.

$$f = \left(\frac{9}{8}\right)^{(T_n-17)} \cdot \left(\frac{10}{9}\right)^{(t_n-12)} \cdot \left(\frac{16}{15}\right)^{(s_n-11)} \cdot A$$

A $Tts()$ value can also be tuned in three-limit (*syntonic becomes Pythagorean comma*) or 53-TET.
Composed music and notation

Step types

**Traditional western music** whole tone and semitone

**Turkish music** *bakiye* (4 commas), *küçük męcennep* (5 commas), *büyük męcennep* (8 commas), *tanini* (9 commas), and *artık ikili* (12 commas)
Making three out of five

Translate Turkish step types to $Ts()$

While tuning Turkish scales the Pythagorean (small) semitone and the augmented second can be obtained as shown below:

$bakiye$ (4 commas) $\frac{T}{s}$

$artık ikili$ (12 commas) $\frac{T \cdot t}{s}$
Making three out of two or one
Translate western step types to $Tts()$

User-defined:
2. $Tts()$ values are mapped to MIDI notes.

Algorithmic:
- Algorithms to translate one- to two-dimensional (*real-time pitch spelling*) and two to three-dimensional data (using *hexachord pattern matching*) are in development.

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Software and implementation

All functions are bundled into a file called *libmuditulib.c*. An implementation for Pure Data is ready yet. A first test will be released during or shortly after the conference. Inclusion of the algorithms can be expected within the next two months.
Software can be found at http://sourceforge.net/projects/muditulib/.
More information can be found at http://muditulib.eu/.