Muditulib

a multi-dimensional tuning library

Funs Seelen

http://muditulib.eu

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Tuning of pitch within the digital domain

1. Tuning theory
2. Music theory: the diatonic scale
3. Multi-dimensional pitch representation systems
4. $Ts()$: a 2-dimensional tuning system
5. $Tts()$: a 3-dimensional tuning system
6. Implementation of the software

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Equal temperament (12-TET)

$$f = 2^{((m-69)/12)} \cdot 440$$

Harmonic spectrum:

partials $1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : etc..$

Intervals:

octave: $2^{(12/12)} = 2$

perfect fifth: $2^{(7/12)} \neq \frac{3}{2}$

major third: $2^{(4/12)} \neq \frac{5}{4}$

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The diatonic scale

Consists of two variable steps:

- The whole tone (T)
- The semitone (s)

The frequency ratio between the eighth and the first note - generally $2:1$ - is called the octave (x).

$$x = T^5 \cdot s^2$$

The diatonic scale

The tonal system used as a starting point is a 7-tone and NOT a 12-tone system.

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Traditional music notation

- Traditional music notation is based on seven notes and the two kinds of steps, and is therefore two-dimensional.
- All intervals in diatonic music are built up from a number of whole tones and a number of semitones.

Two-dimensional representation of western note naming system.

One horizontal step is one whole tone, one vertical step a semitone.

Fbb	Gbb	Abb	Bbb	Cb	Db
Ebb	Fb	Gb	Ab	Bb	C
Db	Eb	F	G	A	B
C	D	E	F#	G#	A#
B	C#	D#	E#	F##	G##

Example intervals

Intervals can be expressed as a list of two values

octave: $T^5 \cdot s^2 \rightarrow (5,2)$

perfect fifth: $T^3 \cdot s \rightarrow (3,1)$

major third: $T^2 \rightarrow (2,0)$

minor third: $T \cdot s \rightarrow (1,1)$

Example pitches

Pitches can be expressed as a list of two values (as a distance to a reference pitch) as well

The reference pitch is set equal to the C corresponding to MIDI note 0. Such a pitch will be called a $Ts()$ value, containing T_n and s_n, the number of whole tones and semitones, respectively.

C,,,, $Ts(0,0)$

C' $Ts(25,10)$

B# (german: H#) $Ts(26,8)$

$Ts()$ values can be easily translated to MIDI notes.

$$m = 2 \cdot T_n + s_n$$

$Ts()$ to note name symbol

The total amount of steps ($T_n + s_n$) defines the character (one of seven ($(T_n + s_n)\%7$)).
Flats or sharps can be extracted from the difference in number between whole tones and semitones.

Obtaining $Ts()$ values

- Directly from the composition (e.g. Lilypond to $Ts()$)
- User-defined: ‘mod’ parameter
- Algorithmic: real-time pitch spelling

The *modulation* parameter

The reference is the standard baroque keyboard setting:

Eb, Bb, F#, C#, G#

Every modulation up replaces one note in the circle of fifths by adding $(1, -2)$ to its assigned Ts value, starting at MIDI note 3 (*E-flat* to *D-sharp*). In the opposite direction it starts at MIDI note 8 (*G-sharp* to *A-flat*). In the current implementation each modulation change is calculated from a default array at 'mod 0'.

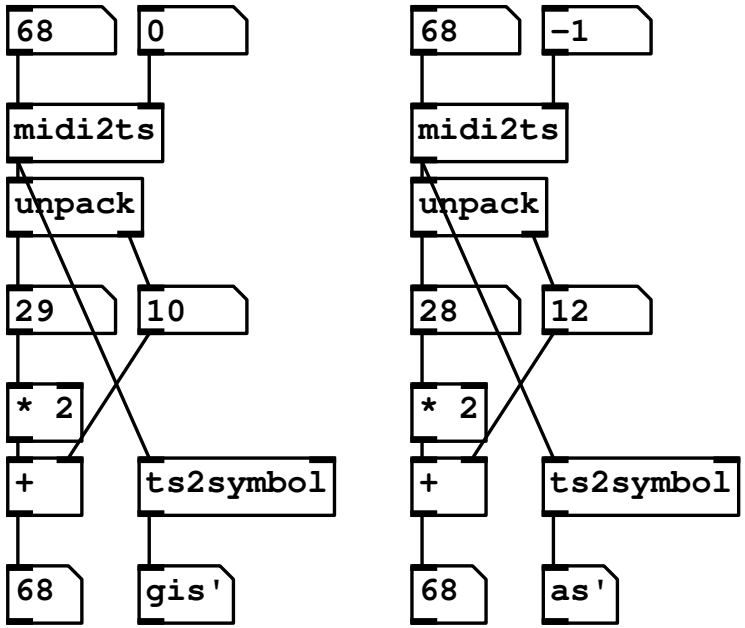

Figure: Examples of [midi2ts] and [ts2symbol] in Pure Data.

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Two-dimensional pitches enable a two-dimensional tuning

Examples of two-dimensional tunings/temperaments are

- Pythagorean
- Mean tone

Temperaments that can be one- AND two-dimensional are the n-TET temperaments.

- 12-TET (2-1)
- 19-TET (3-2)
- 31-TET (5-3)
- 53-TET (9-4; can even be three-dimensional (Tts, 9-8-5))

The type of a two-dimensional tunings can be defined by ratio r.

r is the semitone to whole tone ratio.

$$r = \frac{\log s}{\log T}$$

Pythagorean (perfect fifth $(3,1)$ and octave $(5,2)$):

- $T = \frac{9}{8}$, $s = \frac{256}{243}$
- $r \approx 0.44247$

$$\frac{3}{2} = 2^{((3+r)/(5+2r))}$$

Other two-dimensional examples

Mean tone (major third $(2,0)$ and octave $(5,2)$): $r \approx 0.60628$

$$\frac{5}{4} = 2^{(2/(5+2r))}$$

n-TET:

- 12-TET $r = \frac{1}{2}$
- 19-TET $r = \frac{2}{3}$
- 31-TET $r = \frac{3}{5}$
- 53-TET $r = \frac{4}{9}$

Calculate $Ts()$ to frequency f

A is the reference frequency at $Ts(29, 11)$ in cycles per second.

$$f = x^{((T_n - 29) + (s_n - 11) \cdot r)/(5 + 2 \cdot r)} \cdot A$$

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Adding a third dimension enables five-limit just intonation and Turkish modes.

The whole tones are then divided into a number of whole tones (T) and some slightly smaller whole tones (t).

Some examples:

octave	$(3,2,2)$
perfect fifth	$(2,1,1)$
major third	$(1,1,0)$
minor third	$(1,0,1)$

$Tts(T_n, t_n, s_n)$

The difference between T and t in five-limit just intonation ($\frac{81}{80}$) is called the *syntonic comma*.
A is the reference at $Tts(17, 12, 11)$.

$$f = \left(\frac{9}{8}\right)^{(T_n - 17)} \cdot \left(\frac{10}{9}\right)^{(t_n - 12)} \cdot \left(\frac{16}{15}\right)^{(s_n - 11)} \cdot A$$

A $Tts()$ value can also be tuned in three-limit (*syntonic* becomes *Pythagorean* comma) or 53-TET.

Composed music and notation

Step types

Traditional western music whole tone and semitone

Turkish music *bakiye* (4 commas), *küçük mücennep* (5 commas), *büyük mücennep* (8 commas), *tanini* (9 commas), and *artık ikili* (12 commas)

Making three out of five

Translate Turkish step types to $Tts()$

While tuning Turkish scales the Pythagorean (small) semitone and the augmented second can be obtained as shown below:

bakiye (4 commas) $\frac{T}{s}$

artık ikili (12 commas) $\frac{T \cdot t}{s}$

Making three out of two or one

Translate western step types to $Tts()$

User-defined:

1. User selects mode and reference MIDI note.
2. $Tts()$ values are mapped to MIDI notes.

Algorithmic:

- Algorithms to translate one- to two-dimensional (*real-time pitch spelling*) and two to three-dimensional data (using *hexachord pattern matching*) are in development.

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Software and implementation

All functions are bundled into a file called *libmuditulib.c*. An implementation for Pure Data is ready yet.
A first test will be released during or shortly after the conference.
Inclusion of the algorithms can be expected within the next two months.

Sources

Software can be found at
`http://sourceforge.net/projects/muditulib/`.
More information can be found at `http://muditulib.eu/`.